Mathematical Modeling

In large part, we use math to model real-world situations and derive predictions based on our models. The first job is to come up with function formulas for different situations.

1. Let's say a rectangle is twice as long as it is wide.
   (a) Let x be the width of the rectangle and y be the length. Write a formula for the area of the rectangle.
   (b) Write an equation relating x and y.
   (c) Use part (b) and plug in to part (a) to get a function that expresses the area of the rectangle as a function of x.
2. Write a function that expresses the area of a circle as a function of its circumference. You can try a similar technique to the last question.
3. Find the length of a side of a square as a function of the length of its diagonal, d.
4. Werizon Vireless offers a plan where customers pay \$10 per month per device, and \$13 per gigabyte of data. ATT&T Vireless offers a plan where customers pay \$20 per month per device and \$9 per gigabyte of data.
   (a) Write a function $W(g)$ expressing the cost per month to a customer as a function of gigabytes used for Werizon Vireless. Assume that partial gigabytes used are prorated.
   (b) Write a function $A(g)$ expressing the cost per month to a customer as a function of gigabytes used for ATT&T Vireless. Assume that partial gigabytes used are prorated.
   (c) Using your functions, what is the interval (of gigabytes used) where a customer would be better off going with Werizon? What about for ATT&T?

(d) S-Mobile Vireless has a plan where customers pay \$5 per month per device, and \$15 per gigabyte for the first 2 gigabytes, and then \$7 per gigabyte after that. Write a function $S(g)$ that expresses the cost as a function of gigabytes used.

(e) For the above functions, what does W^{-1}, A^{-1}, and S^{-1} represent? Joe Schmo calculated $W^{-1}(50), A^{-1}(50)$, and $S^{-1}(50)$. Why?

5. The population of cute furry mice in Storrs over time is given by $p(t) = \dfrac{2100}{1+8e^{.9t}}$, where t represents months.
   (a) Is the population increasing or decreasing or both?

   (b) As $t \to \infty$, $p(t) \to$ ______.

   (c) Interpret your result from part (b). Does this model make sense? Explain.

6. The profit from selling fried crickets to college students is given by $P(c) = -c^2 + 200c - 1800$, where c is the number of buckets of crickets sold. A bit of terminology is in order. To run the cricket business, you have *fixed costs* that don't depend on the number of crickets you fry (e.g., you have to buy the fryer, set up your cricket stand), and you have *variable costs* that are based on the number of crickets you sell (e.g., you have to buy crickets to fry). The *total cost* is the cost to produce c crickets, so that is the sum of the fixed and variable costs. The *marginal cost* you can take to be the cost to produce "1 more cricket" than you already are. Finally, the *break-even point* is the number of crickets at which you're not making money and you're not losing money.
   (a) What is $P(0)$, and what does that represent?

   (b) Where does $P(c) = 0$, and what does that represent?

(c) How many crickets should you sell to maximize your profit, and how much profit can you make?

7. A rectangular package sent through the US Postal Service must satisfy that the length plus the girth is less than 108 inches (the girth is the distance around the box). Let's assume that the box we have has a square base, so the two shorter sides have the same measurement.

   (a) Draw a picture to represent this situation. Let ℓ represent the length of the box and x represent the length of the shorter sides. Label your diagram.

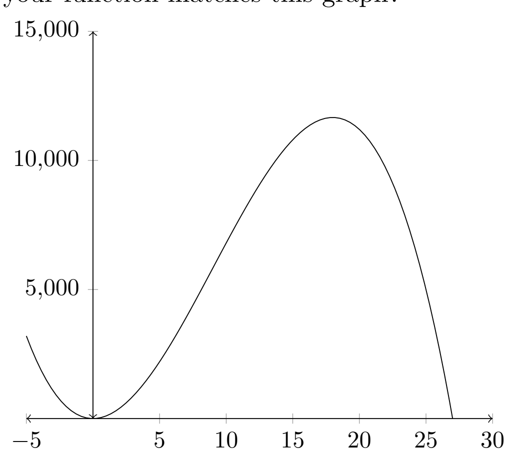
   (b) Write a formula for the girth of the box.

   (c) If you have a box with maximum length and girth, write an equation that is satisfied by this box involving ℓ, x, and the number 108.

   (d) Write a formula for the volume of the box involving ℓ and x.

   (e) Use your answer from part (c) (perhaps with a little algebra work) and substitution so that you get a function $V(x)$ for the volume of the box in terms of x.

   (f) Your function should have a graph that looks like the one below. How could you check if your function matches this graph?



(g) Using the graph, for approximately what value of x will $V(x)$ be maximized?

(h) What are the rough dimensions of the largest (in volume) box you can send through the USPS?

8. In the decibel scale, the Sound Intensity Level L (measured in decibels) of a sound intensity I (measured in watts per square meter) is given by $L(I) = 10\log\left(\frac{I}{10^{-12}}\right)$. For example, a sound intensity of 10^{-12} is just at the threshold of human hearing, and the decibel level of that is $L(10^{-12}) = 10\log\left(\frac{10^{-12}}{10^{-12}}\right) = 0$ decibels.

(a) How many decibels is a sound intensity of 10^{-4}?

(b) In 2013, the crowd at a Seattle Seahawks game broke the world record for the largest ever decibel noise at a sports stadium, reaching 137.6 decibels. What was the sound intensity?

(c) A jackhammer weighs in at about 100 decibels. At about 140 decibels (this is about the intensity on the deck of an aircraft carrier), humans can experience hearing loss with even short-term exposure. How many times more intense would the sound intensity be at 140 decibels compared to 100 decibels?

A. 1.4 times
B. 140 times
C. 1000 times
D. 1400 times